Non Catastrophic Endogenous Growth with Pollution and Abatment

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Abstract:

When there are pollution externalities the competitive equilibrium is not Pareto-optimal nor environmentally sustainable even if abatement activities are endogenously decided. In this paper we introduce the possibility of an ecological catastrophe like the one predicted by the global climate change, imposing the constraint of an upper-limit to the pollutants stock. We characterize the socially optimal solution and study conditions for the sustainability of the balanced growth path. We find a trade-off between environmental quality and growth. The rate of growth depends negatively on the weight of environmental care in the utility function and positively on the population growth rate. We show that the emissions reduction recommended in the Kioto protocol is an appropriate policy to avoid the ecological catastrophe and ensure global efficiency and positive long-run growth.

Keywords: Environment, Externalities, Optimal Growth, Ecological Catastrophe, Sustainability.

JEL classification: C61, C62, O41, Q5.

1 Introduction

In a recent work on endogenous growth theory and the environment [Smulders (1999)], we can read that "*Many other models only incorporate a flow variable to represent the environment. Thus ignoring the accumulation of wastes and the irreversibility of environmental damage, these models are not able to examine the possible conflict between short-run and long-run consequences of economic growth on the environment, but they prove to be a useful simplification to examine, for instance, the effects of different environmental tax issues*". The lesson is that the stock of accumulated pollutants has to be explicitly incorporated into models when we study the issue of long-run growth sustainability. In such a case, if pollution increases with economic growth, it may happen that growth ceases when the stock reaches a certain upper-bounding level. Moreover, long-run sustainability depends not simply on the level of emissions but also on the assimilative capacity of the environment. Indeed, as López (1994) points out, the world's capacity to absorb pollution is limited and once pollution stock approaches the absolute tolerable limit, economic growth would not become feasible anymore because the economy will be falling down into an extreme situation of catastrophic state.

The previous point leads us thinking that it is of great relevance to ask whether there are limits to growth. The global economic collapse is more probable that arises in models of endogenous growth, in which the economy follows a path with sustained long-run growth, if pollution emissions appear positively related to the economic activity (production and consumption) as a by-product [Gradus and Smulders (1993), Ligthart and Ploeg (1994), Michel and Rotillon (1995), Mohtadi (1996)]. In such a case, if pollution tolerance is limited it will be attained the state of ecological catastrophe that represents an effective and absolute limit to growth.

This problem, however, may be mitigated when it is possible for economic agents to undertake emissions abatement activities or control for the degree of pollution associated with production technologies [Gradus and Smulders (1993), Ligthart and Ploeg (1994), Byrne (1997), Stokey (1998), Andreoni and Levinson (2001), Reis (2001)]. Actually, pollution stocks can be diminished not only by increasing the regenerative capacity or by reducing the polluting activity, but also by means of pollution abatement actions that contribute to determine the degree of dirtiness associated with technology as well as the net flow of pollutants to the environment. However, improving environmental quality requires abatement expenditures that leave less resources available for growth-oriented investment activities. Hence, a trade-off between production growth and environmental quality has to be featured in this enlarged framework.

In general, under the existence of environmental externalities, it is expected to find lower rates of growth for output and pollution when pollution is optimally controlled.[1] In this sense, the opportunity for pollution abatement generates a mechanism that may act as a limit to growth, although less strong than the previous one. According to this, the important question is whether economic growth and environmental protection are reconcilable. That is, whether optimal sustained growth is compatible with ecological sustainability of the economy as a whole. Moreover, the query on whether environmental concern will eventually limit growth has to be answered looking at two different issues: first, the effects of pollution abatement on the long-run rate of growth; and second, the evolution of the stock of pollutants with respect to the ecological catastrophic upper-limit.

All these questions will be analyzed more accurately in this paper in a simple model of endogenous growth. Given that we are not directly interested on how technological change has been originated, but on conditions under which sustained endogenous growth and ecological sustainability are compatible, our model builds upon the traditional Rebelo's (1991) one-sector AK model to which we incorporate pollution. Moreover, this model is also appropriate because of its simplicity, which allows us to focus on the performance of developed economies and their outcomes in the long run, the period in which sustainability appears as a relevant issue.

[1] Things could be different if a pollution externality on the side of production is considered [Gradus and Smulders (1993), Ewijk and Wijnbergen (1995), Mohtadi (1996), Smulders and Gradus (1996)]. In such a case, environmental quality changes production opportunities by affecting the economy's productivity, and an increase in environmental care may boost growth.

Welfare depends on consumption but also on the quality of the environment where agents consume. In this model pollution arises from production and enters the consumer's utility function playing the role of a local externality. This externality is explained by the existence of numerous agents who take into account the local effect which pollution flow exerts on their respective utilities, without having any influence on the generating process which mainly depends on firms. One central aspect of the analysis below is the explicit consideration of abatement activities. These are costly because they absorb resources reducing investment and consumption possibilities. In this setting households show environmental concern but they do not decide on abatement while firms, which take into account the cost of such an activity, do not receive the corresponding benefits.[2]

Moreover, there is another externality which arises from the fact that people take into account the local negative effect from the flow of pollution, but they are not aware of the global negative effect from an eventual ecological catastrophe. This is an aggregate welfare externality because individual agents decide on the emissions flow but do not control for the accumulated stock of pollutants. However, the aggregate stock will have a drastic effect on the individual agent's welfare at the moment of economic collapse. This matter shows a strong parallelism with the real problem of the enhanced greenhouse effect caused by anthropogenic pollution emissions. Economic activity releases greenhouse gases into the atmosphere. To a certain extent they are removed by sinks but one important part of them are stored in the atmospheric reservoir. This increases the greenhouse effect which results in an additional global warming of the Earth's surface and atmosphere. Once the accumulated stock of greenhouse gases in the atmosphere goes beyond a critical level, it may adversely affect ecosystems causing a drastic climate change. Face to this problem, economic agents are concerned about the local effects of pollution but they do not consider the global effect for decision making. In this context, the Kyoto protocol makes evident the previous aggregate externality problem and implements policies and measures which

[2]We ignore here any other local externality associated with the flow of pollution, which could play a significant role by affecting the productivity of factors via the health of workers or the quality of inputs.

try to control and reduce emissions flows.[3]

Because of these two environmental externalities the competitive equilibrium does not work well. On the one hand, the equilibrium path is not Pareto-optimal and, on the other hand, this path leads the economy to the state of ecological catastrophe. Consequently, we will study the opportunities for an efficient management of the economy with special attention to the potential risk of an environmental and economic collapse. Efficiency, as traditionally assumed, is not sufficient for sustainability but, as we will show, Pareto optimality is necessary to produce sustainable outcomes. Moreover, along the article we are going to answer the usual questions: *(i)* Do environmental externalities influence growth? *(ii)* What are conditions for sustained balanced growth when environment matters? *(iii)* Which is the effect of environmental concern on the rate of growth? *(iv)* Will pollution controls and abatement reduce growth rates? *(v)* Under what conditions is sustainability feasible?

The article is organized as follows. Section 2 describes the economy and introduces the assumptions featuring a general equilibrium one-sector endogenous growth model in which pollution is a by-product of economic activity, but it may be reduced by spending a fraction of the aggregate output on abatement. In section 3 we briefly study the decentralized competitive equilibrium without regulation. In sections 4 and 5 we study the socially optimal solution assuming sufficient conditions for interior solutions. Using the unconstrained trajectories, we characterize growth in the social optimum and analyze under what conditions sustained balanced growth is feasible. Section 6 focuses on ecological sustainability and non-catastrophic growth, with special attention to conditions which ensure them. There, we solve the general model allowing for corner solutions and study how, if pollution stock reaches the upper-limit, the central planner could change the value of the dirtiness index. We also characterize growth in the aftermath and compare with the previous one. Finally, section 7 summarizes and concludes.

[3] The Kioto protocol establishes the necessary cooperating context where the adverse consequences of this second externality may be solved and sustainability guaranteed.

2 The economy

2.1 Production

The model economy is a one sector closed economy. Output is obtained according to an aggregate production function of the AK type where capital is the only factor needed to produce,

$$Y(t) = AK(t). \tag{1}$$

In this model K is an aggregate composite of different sorts of capital which, in a broad sense, includes physical as well as human capital. For the sake of simplicity, we assume that this production function arises from the direct summation of the individual production functions for many identical firms.

2.2 Pollution and abatement

One feature of this model, absent from the canonical endogenous growth AK model, is the existence of a stock of pollutants S that is increased by polluting activities such as production Y, and is reduced by abatement $\mathcal{B}$ as well as by the corresponding natural regeneration at a constant rate $\delta > 0$.[4] Moreover, it is assumed an upper-limit for S, called $S^{\max}$, which plays the role of a critical value for which if the current stock goes beyond it, a catastrophic state is reached in the economy. Under these assumptions, *sustainable development* will be characterized as a situation where the main economic variables show positive long-run balanced growth while, at the same time, they contribute to generate an accumulated stock of pollutants smaller than (or equal to) the critical value $S^{\max}$.

The above-mentioned abatement effort $\mathcal{B}$, which is costly and endogenously decided by agents, will be measured in terms of output $Y(t)$ in such a way that these two variables

[4] An alternative to this constant exponential rate of pollution decay, which implies that natural regeneration is a linear function of the pollution stock, is the inverted U-shaped decay function modelized by Tahvonen and Withagen (1996) and Tahvonen and Salo (1996). This one implies that a pollution stock level that is sufficiently high will reduce the rate of natural regeneration to zero.

relate to each other according to

$$\mathcal{B}(t) = Y(t) - Y_N(t) = (1 - z(t))\, Y(t). \tag{2}$$

Here $z(t)$ represents, as in Stokey (1998) and the opposite to Reis (2001), a measure of the effective dirtiness of the technique used to produce. Obviously, $z(t) = 1 - \frac{\mathcal{B}(t)}{Y(t)} \in [0,1]$ because resources devoted to clean pollution could never pass the upper bound established by current production. Therefore, any choice for z close to zero or one automatically makes the existing technique less or more polluting respectively. The above expression introduces a definition for the net output as

$$Y_N(t) = z(t)Y(t). \tag{3}$$

The equation governing the motion of S may be written as $\overset{\bullet}{S}(t) = P(Y(t), \mathcal{B}(t)) - \delta S(t)$, where $P(Y(t), \mathcal{B}(t))$ represents the emissions flow associated with the endogenously determined levels of polluting and abatement activities. This flow is increasing with respect to Y and decreasing with respect to $\mathcal{B}$, i.e. $P_1 > 0$ and $P_2 < 0$. Function $P(.)$ is assumed homogeneous of degree zero, i.e. an equal proportional increase in both output and abatement leaves the emissions flow unchanged. Consequently, the emissions flow may be rewritten as $P(Y(t), \mathcal{B}(t)) = E\left(\frac{\mathcal{B}(t)}{Y(t)}\right)$, where we assume strict concavity: $E' < 0$, $\lim_{x \to 0^+} E' < 0$, $-\infty < \lim_{x \to 1^-} E' < 0$, $E'' < 0$, $E(0) = E^M > 0$ and $E(1) = 0$. Actually, E^M represents an effective upper bound for the emissions function, which is high enough to conduct the economy, if it prevails, to the state of ecological catastrophe.[5] Now, substituting the previous variable transformations into the differential equation for the motion of the stock of pollutants we get

$$\overset{\bullet}{S}(t) = E(1 - z(t)) - \delta S(t), \tag{4}$$

where $E_z = -E' > 0$, $0 < \lim_{z \to 0^+} E_z < +\infty$, $\lim_{z \to 1^-} E_z > 0$ and $E_{zz} = E'' < 0$. Taking as reference $z = 1$, which implies that no abatement effort is done and that emissions

[5] Tahvonen and Salo (1996) conceives this upper bound as the emissions level for which residents decide to move to other locations. On the other hand, we can interpret these emissions as a maximum level beyond which production starts to be delocalized by transferring abroad the polluting technology.

flow reaches the maximum level E^M, the larger the reduction in z the more effective the reduction in emissions. Or, put in other words, as long as we produce with a cleaner technology, the effectiveness measured in terms of emissions reduction of any additional pollution abatement that reduces z, will be larger.[6]

2.3 Investment

According to the aggregate resources constraint, net output may be devoted to consumption or capital accumulation. For the sake of simplicity we do not consider capital depreciation. Hence, net investment equals gross investment and the capital stock is governed by the differential equation

$$C(t) + \overset{\bullet}{K}(t) = Y(t) - \mathcal{B}(t). \tag{5}$$

This equation also reflects the cost of the abatement activity in a very simple way: one additional unit of abatement effort is automatically 'transformed' into a lower unit of output available for consumption or capital accumulation. This particular 'one-to-one' transformation contributes to simplify our analysis.

2.4 Preferences

The economy is populated by many identical and infinitely lived agents. Population, denoted by N, is assumed to be growing at a constant rate $0 < n < A$. The initial population $N(t_0)$ is normalized to one. Individual preferences are assumed to be represented by a twice continuously differentiable instantaneous utility function $V(c(t), P(t))$, which depends positively on the current per capita consumption c and negatively on the emissions flow P [Gradus and Smulders (1993), Ligthart and Ploeg (1994), Selden and Song (1995), Reis (2001)]. Under this assumption, households do not take care for the stock of

[6] There is not empirical evidence about this assumption and, moreover, even if we assume convexity, given the nature of the model, the results below do not change. Instead, the concavity assumption allows for a huge simplification of all subsequent analysis concerning sufficient conditions for optimality and the convergence of integrals.

pollutants in the environment, but only for the current flow of polluting emissions. This is because the local stock effect of pollution is assumed short-lived and the abatement activity, which reduces emissions and facilitates regeneration, makes the local stock effect negligible.[7]

As we have previously shown, emissions depend positively on production and negatively on pollution abatement, two variables that appear related to each other according to (2). Moreover, the definition of the emissions function establishes a monotonic decreasing relationship between the emissions flow and the ratio abatement to output. Hence, we may consider the abatement effort relative to the economy's dimension as the second argument in the utility function, which means that households derive utility directly from the aggregate effort addressed to reduce pollution emissions. That is, utility indirectly depends on environment quality, which is increased by abatement and reduced by production. In this model, moreover, we find that the dimension of the economy may be measured by output as well as by capital stock, given the linear form of the production function. Therefore, the instantaneous utility function may be written as $U\left(c(t), \frac{\mathcal{B}(t)}{Y(t)}\right)$ with $U_c > 0$ and $U_2 > 0$. Or, given that the relationship between abatement and production allows for the substitution $\frac{\mathcal{B}}{Y} = 1 - z$, also as $U\left(c(t), z(t)\right)$ with $U_z = -U_2 < 0$.[8] Moreover, we assume decreasing marginal utilities: $U_{cc} < 0$ and $U_{zz} = U_{22} < 0$, as well as strict concavity with respect to both arguments taken together, $U_{cc}U_{22} - \left(U_{c2}\right)^2 > 0$.

The structure of the model allows for the existence of a long-run balanced growth path,

[7] This also implies that we ignore global stock effects in the representation of households' preferences. An important stream of literature considers that welfare depends on the stock of pollution rather than on the current flow [Huang and Cai (1994), Mohtadi (1996), Tahvonen and Salo (1996), Byrne (1997), Kelly (2003)]. However, if the flow of pollution is increasing with production, then capital accumulation that increases future output also increases future flows of pollution. Hence, we find a general consensus in the literature [Gradus and Smulders (1993), Smulders and Gradus (1996), Aghion and Howitt (1998), Reis (2001)] according to which, in the context of this model, if we consider the stock of pollution as an argument in the utility function, we will obtain the same fundamental results but at the cost of a more complex analysis.

[8] In short, given that P is an increasing monotonous transformation of z, the two ordinal utility functions represent the same preference ordering.

defined as an allocation in which consumption per capita grows at a constant rate and the dirtiness index is constant. To ensure that such a path may exist in this model it is necessary to assume that the particular instantaneous utility function is multiplicatively separable and of the CIES form [King, Plosser and Rebelo (1988), Bovenberg and Smulders (1995; 1996), Smulders and Gradus (1996), Ladrón de Guevara et al. (1999)] [9]

$$U\left(c(t), z(t)\right) = \frac{c(t)^{1-\Phi}}{1-\Phi}\left(1 - z(t)\right)^{\alpha(1-\Phi)}. \tag{6}$$

In this function, the parameter that represents the relative weight of environmental care in utility is assumed to be positive and lower than one, $0 < \alpha < 1$, and the inverse of the constant intertemporal elasticity of substitution is allowed to be lower or greater than one, $0 < \Phi \lesseqgtr 1$. The previous utility function fulfill all the above mentioned assumptions concerning first and second derivatives. The strict concavity assumption requires as sufficient condition that the determinant of the Hessian matrix be positive, which implies the additional parameter constraint $\Phi > \frac{\alpha}{1+\alpha}$.[10]

3 The competitive solution

First of all, we will consider the structure of this economy from the point of view of the non-regulated competitive equilibrium. In this economy, assuming that there are no

[9] Bovenberg and Smulders (1995) also considers additional restrictions on ecological relationships and technology. With respect to the first, we remind that hereafter we are going to study conditions for ecologically non-catastrophic states. With respect to the second, we have to recall that our one sector and one accumulable factor model builds upon a linear production function which summarizes the whole set of technological requirements postulated by these authors.

[10] Environmental literature has long dealt with the sign of the second order cross derivative of the instantaneous utility function [Michel and Rotillon (1995), Mohtadi (1996)]. In our case $U_{c2} = -U_{cz} = \alpha(1-\Phi)c^{-\Phi}\left(1-z\right)^{\alpha(1-\Phi)-1}$, which is negative (positive) as long as Φ is greater (smaller) than one. That is, as long as the intertemporal elasticity of substitution is smaller (greater) than one. Empirical evidence seems to corroborate the case of a low intertemporal elasticity of substitution and, hence, $U_{c2} < 0$. This implies that the marginal utility of consumption decreases as the environmental quality increases. Namely, consumption and pollution are complements in terms of preferences. However, the model works exactly the same in the opposite case in which $U_{c2} > 0$.

depreciation charges, each competitive firm faces the following stationary optimization problem, given the absence of adjustment costs and any other intertemporal element in its present value maximization problem,

$$\max_{\{K_i, z_i\}} \Pi_i = Y_i - rK_i - \mathcal{B}_i$$

$$\text{s.t.} \quad 0 \leqslant z_i \leqslant 1,$$

$$Y_i = AK_i,$$

$$\mathcal{B}_i = (1 - z_i) Y_i,$$

$$K_i > 0.$$

The first order conditions are

$$r = Az_i, \tag{7}$$

$$0 = (1 - z_i) AK_i. \tag{8}$$

From (8), because of the slackness condition, we observe that it is optimal from the point of view of the individual firm to fix $z_i = 1$ and then, by (7), we get $r = A$. These results imply zero quasi-rents at the maximum. But this also means that individual firms have no incentive to allocate resources to pollution abatement because of the externality originated in the conflict between the private nature of the cost of this activity and the social nature of its benefits, which are beyond the firm's control. Consequently, in the competitive equilibrium we will observe that production is undertaken by firms with the most polluting of the available production techniques.

Instead, households preferences are sensitive to the pollution emissions flow as has been represented in their utility functions. They show a clear preference for reducing z below unity according to the assumption $U_z < 0 \ \forall z$. We don't need to explicitly solve the optimization problem for households. Actually, given the form of the utility function households will never choose such an extreme value for z, and this will generate a fundamental market mismatch between abatement demand and supply that exactly reflects the consequences of the above-mentioned externality. There is no incentive for agents to internalize the negative external effect and, consequently, the equilibrium path is not Pareto optimal.

On the other hand, although it has not been included as an argument in the utility function, we also have to consider in our analysis the evolution of the aggregate stock of pollutants S because it is crucial from the point of view of the sustainability of the long-run economy's aggregate outcomes. However, this is not controlled by firms or households individually because both take as given the level of this stock. In fact, the competitive equilibrium that leads the firm to choose the dirtiest technique to produce, $z = 1$, has dreadful implications with respect to the sustainability problem. Consider the equation governing the motion of the aggregate stock of pollutants under such an extreme value $\overset{\bullet}{S}(t) = E^M - \delta S(t)$, which has the solution $S(t) = \left(S_0 - \frac{E^M}{\delta}\right) e^{-\delta(t-t_0)} + \frac{E^M}{\delta}$. Then S monotonically increases converging to the value $\frac{E^M}{\delta} > S^{\max}$, which means that eventually the state of ecological catastrophe will be reached.

In conclusion, the presence of a welfare pollution externality in a decentralized competitive economy, which imply not much abatement and too much pollution, calls for some sort of public intervention. Without any corrective environmental policy the environment will be damaged up to the level of irreversible catastrophe, and sustained growth, if there exists, will not be sustainable.

4 Optimality conditions

Now, we will focus on the socially optimal solution for the model economy described in previous sections.[11] As we have shown, there are two externalities connected with the environmental problem: one local and the other aggregate. The first one conforms to the traditional view according to which an externality is present whenever the welfare of

[11] The matter of how to replicate the efficient path in a competitive economy by means of an optimal environmental policy, is beyond the scope of this paper. However, the reader will find in Mohtadi (1996), Smulders and Gradus (1996), as well as in Rubio and Aznar (2002) a discussion of how the government can implement pollution charges, emission standards and public abatement, which allow the competitive economy to generate efficient outcomes in an endogenous growth model with an AK technology.

some agents (households) depends on variables whose values are chosen by others (firms). Instead, the second one works like an extreme welfare (technological) externality because individual decisions on pollution emissions do have uncontrolled effects on the aggregate stock of pollutants, which in turn will have a drastic effect on the agent's welfare from the moment of the ecological catastrophe (economic collapse). The socially optimal solution simultaneously internalizes both externalities. First, by considering the total costs and benefits from pollution abatement and, second, by taking into account the global negative effect from an eventual ecological catastrophe associated with the accumulated stock of pollutants. In this setting, the central planner looks at the social welfare, which is defined as

$$W = \int_{t_0}^{\infty} \frac{c^{1-\Phi}}{1-\Phi} (1-z)^{\alpha(1-\Phi)} e^{-(\rho-n)(t-t_0)} dt$$

if $S(\tau) \leqslant S^{\max} \quad \forall \tau$, or

$$W^c = \int_{t_0}^{t^c} \frac{c^{1-\Phi}}{1-\Phi} (1-z)^{\alpha(1-\Phi)} e^{-(\rho-n)(t-t_0)} dt$$

if $S(\tau) > S^{\max} \quad \forall \tau > t^c$, where $t^c < \infty$ represents the period in which occurs the ecological catastrophe and the corresponding economic collapse.

Assuming that the central planner is intended for maximizing social welfare, the way he can reach this target implies to maximize W subject to the constraint $S(t) \leqslant S^{\max}$. However, in this section we shall study interior solutions alone. Therefore, although the dynamic optimization problem has to be formulated introducing as an explicit constraint the no-catastrophe condition, which implies that the central planner takes care of trajectories leading to catastrophic states and optimally decides to avoid them by choosing the controls appropriately, we leave such a general procedure for a next section. For now, we specify the optimization problem without this state constraint that applies throughout the planing period, but take into account the evolution of the aggregate stock of pollutants in the environment.[12] Accordingly, we will obtain unconstrained optimal trajectories for

[12] We do that in line with the recommendation of Chiang (1992): "*Although we cannot in general expect the unconstrained solution to work, it is not a bad idea to try anyway. Should that solution turn out to satisfy the constraint, then the problem would be solved. Even if not, useful clues will usually emerge*

which we are going to check below whether they are ecologically sustainable or not. In particular, we will show sufficient conditions on parameters that ensure the sustainability of such trajectories. Moreover, a similar decision is made here with respect to the static control constraints $0 \leqslant z \leqslant 1$. At this stage we ignore these constraints but, later on, we will also check them for the solution trajectories. This will lead us to identify two parameter conditions which make the control variable bounded.

Under these premises, assuming interiority with respect to the static constraints, the planner's problem consists in choosing the sequence $\{c(t), z(t), t \geq t_0\}$ which, for a given positive social rate of discount $\rho > n$, solve the optimization problem

$$\max_{\{K,S,c,z\}} \int_{t_0}^{\infty} \frac{c^{1-\Phi}}{1-\Phi} (1-z)^{\alpha(1-\Phi)} e^{-(\rho-n)(t-t_0)} dt$$

$$s.t. \qquad (1)\text{-}(5),$$

$$\text{for } k(t_0) = k_0 > 0 \text{ and } s(t_0) = s_0 > 0 \text{ given.}$$

From now on we will use lowercase letters to represent variables in per capita terms. The current value Hamiltonian is

$$\underset{\{c,z,q,k,\mu,s\}}{H^c} = \frac{c^{1-\Phi}(1-z)^{\alpha(1-\Phi)}}{1-\Phi} + q[Akz - c - nk] + \mu[e(1-z) - (\delta+n)s],$$

where q and μ are the co-states for k and s, respectively, and represent their corresponding shadow prices. The first order necessary conditions are

$$q = c^{-\Phi}(1-z)^{\alpha(1-\Phi)}, \tag{9}$$

$$q + \mu\frac{e_z}{Ak} = \frac{\alpha c^{1-\Phi}(1-z)^{\alpha(1-\Phi)}}{Ak(1-z)}. \tag{10}$$

As we have seen, gross product may be allocated to consumption, investment or abatement. On the margin, according to (9), goods must be equally valuable if they are consumed or accumulated as new physical capital. Namely, the marginal utility of consumption today must be equal to the marginal shadow value of physical capital (consumption tomorrow). According to (10), at equilibrium the implicit price of a more dirty

regarding the nature of the true solution." For a complete formulation of the optimal control problem, including every relevant constraint, the reader may look at section 6.1 and the Appendix.

technique, that is, the value measured in units of utility of the net marginal product from a more dirty technique (qAk) plus the shadow value of the marginal emission associated with such a more dirty technique (μe_z), must be equal to the marginal utility of a cleaner one. Namely, the entire valuation of a marginal reduction in resources devoted to abatement, which contributes to increase consumption (present or future) as well as the stock of pollutants, must be equal to the marginal utility of those resources when devoted to abatement, which contribute to increase environmental quality. Moreover, the dynamic conditions for quantities and shadow prices are

$$\overset{\bullet}{k} = Akz - c - nk, \tag{11}$$

$$\overset{\bullet}{q} = \rho q - Azq, \tag{12}$$

$$\overset{\bullet}{s} = e(1 - z) - (\delta + n)\, s, \tag{13}$$

$$\overset{\bullet}{\mu} = (\rho + \delta)\, \mu, \tag{14}$$

together with initial conditions k_0 and s_0 and the transversality conditions

$$\lim_{t \to \infty} e^{-(\rho - n)(t - t_0)} qk = 0, \tag{15}$$

$$\lim_{t \to \infty} e^{-(\rho - n)(t - t_0)} \mu s = 0. \tag{16}$$

Before the resolution of the whole dynamic system, we can study the block of equations related to the stock of pollutants and its shadow price. Looking for that, we first resolve equation (14): $\mu\,(t) = \mu\,(t_0)\, e^{(\rho+\delta)(t-t_0)}$. Then, we integrate (13): $s\,(t) = s_0 e^{-(\delta+n)(t-t_0)} + \int_{t_0}^{t} e(1-z\,(\tau))e^{-(\delta+n)(t-\tau)} d\tau$. Finally, if we substitute both into the transversality condition (16), the result is $\lim\limits_{t \to \infty} \mu\,(t_0) \left\{ s_0 + \int_{t_0}^{t} e(1 - z\,(\tau))e^{(\delta+n)(\tau - t_0)} d\tau \right\} = 0$. This condition holds if, and only if, $\mu\,(t_0) = 0$ because the integral on the r.h.s. cannot be negative. Consequently, the above-mentioned solution to (14) determines that, $\forall t \geq t_0$,

$$\mu\,(t) = 0. \tag{17}$$

This implies that even when the central planner internalizes the costs and benefits of abatement activity, and takes into account the gradual evolution of the aggregate stock of pollutants in the environment, he optimally assigns zero value to the social shadow price of the stock of pollutants. This is so because, although the central planner ensures an efficient resource management in the economy, his main goal consists in maximizing social welfare and, since we are studying interior solutions, the state constraint involving the stock of pollutants has been left out. That is, the potential impact of a drastic ecological catastrophe has not been taken into account.

In this case (17), (9) and (10) imply the tangency condition

$$z = 1 - \frac{\alpha}{A}\frac{c}{k}, \tag{18}$$

as well as the two control functions

$$c = c\,(k, q) = \left(\frac{\alpha}{A}\right)^{\frac{\alpha(1-\Phi)}{\Phi-\alpha(1-\Phi)}} q^{\frac{-1}{\Phi-\alpha(1-\Phi)}} k^{\frac{-\alpha(1-\Phi)}{\Phi-\alpha(1-\Phi)}}, \tag{19}$$

$$z = z\,(k, q) = 1 - \left(\frac{\alpha}{A}\right)^{\frac{\Phi}{\Phi-\alpha(1-\Phi)}} q^{\frac{-1}{\Phi-\alpha(1-\Phi)}} k^{\frac{-\Phi}{\Phi-\alpha(1-\Phi)}}. \tag{20}$$

Substituting (19) and (20) into (11) and (12), we get the dynamic system

$$\overset{\bullet}{k} = (A-n)\,k - A^{\frac{-\alpha(1-\Phi)}{\Phi-\alpha(1-\Phi)}} \left[\alpha^{\frac{\Phi}{\Phi-\alpha(1-\Phi)}} + \alpha^{\frac{\alpha(1-\Phi)}{\Phi-\alpha(1-\Phi)}}\right] q^{\frac{-1}{\Phi-\alpha(1-\Phi)}} k^{\frac{-\alpha(1-\Phi)}{\Phi-\alpha(1-\Phi)}}, \tag{21}$$

$$\overset{\bullet}{q} = (\rho - A)q + A^{\frac{-\alpha(1-\Phi)}{\Phi-\alpha(1-\Phi)}} \left[\alpha^{\frac{\Phi}{\Phi-\alpha(1-\Phi)}}\right] q^{\frac{-(1-\Phi+\alpha(1-\Phi))}{\Phi-\alpha(1-\Phi)}} k^{\frac{-\Phi}{\Phi-\alpha(1-\Phi)}}, \tag{22}$$

with the initial condition $k\,(t_0) = k_0$ and the transversality condition (15). These two differential equations conform a non-linear dynamic system, which has a particular structure that makes it susceptible of being solved in closed form. Applying the method developed in Ruiz-Tamarit and Ventura-Marco (2004), particularly *Propositions 1* and *2*, we conclude that it does exist a unique optimal solution trajectory for $k(t)$ and $q(t)$ with the closed form representation

$$k(t) = k_0 \exp\left\{\frac{A - \rho - \alpha(\rho - n)}{\Phi - \alpha(1-\Phi)}(t - t_0)\right\}, \tag{23}$$

$$q(t) = q(t_0) \exp\left\{-\Phi\frac{A - \rho - \alpha(\rho - n)}{\Phi - \alpha(1-\Phi)}(t - t_0)\right\}, \tag{24}$$

$$q(t_0)^{\frac{1}{\Phi-\alpha(1-\Phi)}} k_0^{\frac{\Phi}{\Phi-\alpha(1-\Phi)}} = \frac{\Phi - \alpha(1-\Phi)}{\rho - A + \Phi(A - n)} \left(\frac{\alpha}{A}\right)^{\frac{\alpha(1-\Phi)}{\Phi-\alpha(1-\Phi)}}. \tag{25}$$

Given the initial capital stock, k_0, equation (25), which arises directly from the transversality condition, gives the initial value for the shadow price, $q(t_0)$. Once the two initial values are known, equations (23) and (24) determine unequivocally the complete trajectories for these two variables. For any $q(t_0)$ other than the one given by (25) the economy places on an explosive trajectory which does not satisfy optimality conditions, in particular the transversality condition. Moreover, given $b_x \equiv \alpha^{\frac{\alpha(1-\Phi)}{\Phi-\alpha(1-\Phi)}} > 0$, the transversality condition holds if, and only if, $a_x \equiv \frac{\rho-A+\Phi(A-n)}{\Phi-\alpha(1-\Phi)} > 0$. This parameter constraint must be satisfied for any positive intertemporal elasticity of substitution, i.e. $0 < \Phi \gtrless 1$, what is not obvious. However, the strict concavity assumption on the utility function imposes the additional parameter constraint

$$\Phi > \alpha(1-\Phi). \tag{26}$$

Hence, the transversality condition (15) holds if, and only if,

$$\rho > A(1-\Phi) + \Phi n. \tag{27}$$

5 Sustained optimal balanced growth

Given (23) and the production function in per capita terms that arises from (1), we obtain

$$y(t) = Ak_0 \exp\left\{\frac{A-\rho-\alpha(\rho-n)}{\Phi-\alpha(1-\Phi)}(t-t_0)\right\}, \tag{28}$$

$$\gamma_y(t) = \gamma_k(t) = \gamma^* = \frac{A-\rho-\alpha(\rho-n)}{\Phi-\alpha(1-\Phi)}. \tag{29}$$

The growth rates of per capita capital stock and output are equal to each other and constant over time along their respective optimal solution trajectories. Using the control functions for consumption and the degree of dirtiness associated with the technique, as given in (19) and (20), we get the optimal solution trajectories for these two variables

$$c(t) = \frac{\rho-A+\Phi(A-n)}{\Phi-\alpha(1-\Phi)}k_0 \exp\left\{\frac{A-\rho-\alpha(\rho-n)}{\Phi-\alpha(1-\Phi)}(t-t_0)\right\}, \tag{30}$$

$$\frac{c(t)}{k(t)} = \left(\frac{c}{k}\right)^* = \frac{\rho-A+\Phi(A-n)}{\Phi-\alpha(1-\Phi)},$$

$$
\begin{aligned}
\gamma_c(t) &= -\frac{\gamma_q(t)}{\Phi} = \gamma^*, \\
z(t) = z^* &= \frac{A\Phi - \alpha\rho + \alpha\Phi n}{A(\Phi - \alpha(1-\Phi))}, \\
\gamma_z^* &= 0.
\end{aligned} \tag{31}
$$

The dirtiness index is expected to be bounded, i.e. $0 \leqslant z^* \leqslant 1$. However, for this to be ensured we need additional parameter constraints. In particular,

$$\Phi(A-n) + n(\Phi - \alpha(1-\Phi)) \geqslant \alpha(\rho - n), \tag{32}$$

$$\Phi(A-n) \geqslant A - \rho, \tag{33}$$

where it may be easily checked that (33) encompasses (27).

These results completely characterize the socially optimal solution. k, c and y grow at the same constant rate; the ratio consumption to capital stock is constant and positive; and the dirtiness index remains fixed forever at a constant value between zero and one. Therefore, the model does not predict transitional dynamics and all the endogenous variables conform a balanced growth path from the beginning.[13]

If we examine a little more the previous results, we find that there could be either positive or negative growth, as well as stationarity. Given (29) and the strict concavity assumption on the utility function, a positive rate of growth $\gamma^* > 0$ arises when $A - \rho > \alpha(\rho - n)$. This condition is compatible with the parameter constraints corresponding to the transversality condition and the lower and upper bounds for z^*, giving

$$\Phi(A-n) + n(\Phi - \alpha(1-\Phi)) > \Phi(A-n) \geqslant A - \rho > \alpha(\rho - n) > 0. \tag{34}$$

[13] The rationale for a constant rate of growth is that the social rate of return to capital is constant. In this model

$$r^* \equiv \frac{\partial Y_N}{\partial K} = Az^* = \frac{A\Phi - \alpha\rho + \alpha\Phi n}{\Phi - \alpha(1-\Phi)},$$

the real return to capital is endogenously determined by preferences and technology parameters. The previous expression also shows that only in the absence of environmental concern, $\alpha = 0$, the interest rate is equal to A, as in the canonical model. Otherwise, it is lower because of the transversality condition (27).

On the other hand, stationarity $\gamma^* = 0$ arises when $A - \rho = \alpha(\rho - n)$, which combined with the remaining parameter constraints leads to

$$\Phi(A-n) + n(\Phi - \alpha(1-\Phi)) > \Phi(A-n) \geqslant A - \rho = \alpha(\rho - n) > 0. \tag{35}$$

In this case all the variables conform a steady state, which is 'chosen' among a multiplicity by the predetermined initial value of per capita capital stock. Finally, although less economically relevant, we could also find negative growth, $\gamma^* < 0$, when $A - \rho < \alpha(\rho - n)$. This is the only case which allows for $A \leqslant \rho$.[14]

The absence of transitional dynamics that makes the short-run dynamics identical to the long-run ones, leads us to undertake the comparative statics analysis for the socially optimal rate of growth and the dirtiness index. The parameter dependences for these two endogenous variables are

$$\gamma^* = \gamma\left(\overset{+}{A}, \overset{-}{\rho}, \overset{-}{\Phi}, \overset{-}{\alpha}, \overset{+}{n}\right), \tag{36}$$

$$z^* = z\left(\overset{+,-}{A}, \overset{-}{\rho}, \overset{-}{\Phi}, \overset{-}{\alpha}, \overset{+}{n}\right). \tag{37}$$

The signs associated with A, ρ and Φ are the usual in the canonical AK model: the larger the capital productivity and the higher the patience of agents, the greater the rate of growth. A newer but very intuitive result is found here: the higher the weight of environmental care in utility the smaller the rate of growth. That is, for higher values of α that imply a higher marginal utility of abatement and a lower rate of return on capital, the central planner optimally decides to devote more resources to abatement and less to capital accumulation and, hence, to growth. However, an striking result arises in this model because of the apparent positive relationship between the rate of growth and the population growth rate. This result is absolutely dependent on the presence of environmental concern in the model, because only in such a case a higher population growth rate leads the central planner to divert resources from abatement and consumption towards capital accumulation. This investment flow is strong enough to compensate for

[14] More technical details about these cases, as well as a complete geometric characterization, may be found in Ruiz-Tamarit and Ventura-Marco (2004).

the new capital requirements due to a greater population, and also to generate a greater rate of growth. Moreover, this effect is stronger as higher is the weight of environmental care in the utility function.[15]

The dirtiness index, in turn, depends positively on the productivity parameter when the intertemporal elasticity of substitution is greater than one, but the sign of this relationship cannot be analytically decided for values of the elasticity lower than one. Moreover, if we find positive growth along the balanced path, the higher the level of patience of agents the higher the value of z. This happens because when consumers are highly patient the central planner optimally decides to reallocate resources towards capital accumulation, which enhance growth, so intensively that even diverts some of the resources previously devoted to pollution abatement, which leads to produce with a more dirty technique. Because of this crowding out effect, the higher the weight of environmental care in the utility function the smaller the dirtiness index. Finally, the greater the population growth rate the higher the dirtiness associated with the effective production technique, which occurs because for higher population growth rates the central planner decides to divert more resources from abatement effort.[16]

These two variables are closely related to each other. Actually, we can make this relationship evident from the first order conditions (11) and (18). If we take the first one and divide by k, and then substitute for the ratio $\frac{c}{k}$ from the second, we get for any $\alpha > 0$

$$\gamma^* = -\left(\frac{A + \alpha n}{\alpha}\right) + \left(\frac{A + \alpha A}{\alpha}\right) z^*, \tag{38}$$

from which we can deduce the following pairs of reference: $(z_1^*, \gamma_1^*) = \left(0, -\frac{A+\alpha n}{\alpha}\right)$ and $(z_2^*, \gamma_2^*) = \left(\frac{A+\alpha n}{A+\alpha A}, 0\right)$. The positive relationship between γ^* and z^* suggests that tighter pollution controls and increased abatement, which reduce the dirtiness index, will have negative effects on the optimal rate of growth. This fact reflects the previous crowding

[15] A similar result, although there it is based on a different explanation, may be found in Bartolini and Bonatti (2003).

[16] The results concerning the population growth rate are consistent with propositions dicussed and tested in Cropper and Griffiths (1994). In that paper, the environment is not a factor that limits productivity as population expands, but a good which quality is degraded by a growing population.

out result according to which greener preferences associated with a shift in preferences towards more environmental concern, i.e. a rise in α, affects negatively both the dirtiness index and the rate of growth.

6 Environmental catastrophe and optimal growth sustainability

One major problem considered in previous models of endogenous growth is the sustainability of the long-run socially optimal and competitive balanced growth paths. This problem has been largely studied in environmental literature where several definitions of sustainability have been proposed [Pezzey (1997), Chichilnisky (1997)]. The most usual concepts of sustainability rely on the intertemporal evolution of utility and consumption, and impose a non-declining trajectory for any of these two variables. In general, sustainability has been conceived as a situation where the needs of the present are satisfied without compromising the needs of the future. This suggests a related problem associated with the valuation of the well-being of present and future generations, and points to the equity condition according to which we must avoid the underestimation of future utilities. In our model, the previous requirements for sustainability are always met because the socially optimal choices guarantee a monotonically increasing profile for consumption and utility. However, although conditions for a positive long-run rate of growth are satisfied, there is a trade-off between growth and environmental quality. Environmental quality is measured by the absence of pollution and increases with abatement but diminishes with production. This trade-off, which results from agent decisions, involve the level of abatement expenditures and, consequently, the value of the dirtiness index.

Sustainability may also be inspected into the model looking at the positive relationship between the dirtiness index and the rate of growth that was shown in (38), which represents in other words the above-mentioned trade-off. As such relationship shows, the more clean the used technology is, the lower the rate of growth. In particular, our model built upon the assumption that people show environmental awareness ($\alpha > 0$) predicts a lower, or at most an equal, rate of growth relative to the rate of growth arising from the canonical model where no environmental concern does exist ($\alpha = 0$). Moreover, as has been shown in the previous section, γ^* may be zero provided that $\alpha = \frac{A-\rho}{\rho-n} > 0$, which is in accordance with the premise that more environmental concern comes in detriment of growth. But this is an extreme case and, for any other value of α smaller than the previous one, we are still allowed to conclude that positive sustained growth and environment preservation may be compatible in the long-run. That is, $0 \leqslant \gamma^* \leqslant \frac{A-\rho}{\Phi}$ as long as $\frac{\rho}{A} \leqslant z^* \leqslant 1$.

Nevertheless, the main question we are going to consider now is whether the socially optimal results from the previous section are compatible with the general condition of no environmental catastrophe, which imposes the constraint that $S(t)$ must be lower than (or equal to) $S^{\max}$. In particular, we have to control at any moment in time the level of $S(t)$ associated with the positive balanced growth path. This approach becomes necessary because along the previous formulation of the central planner's optimization problem we did not take explicitly into account the inequality constraint $S(t) \leqslant S^{\max}$. Actually, none of the previously studied results give rise to explicit constraints on the level and dynamics of $S(t)$.

The equation governing the dynamics of the stock of pollutants is $\overset{\bullet}{S}(t) = E(1 - z(t)) - \delta S(t)$, with $0 < S_0 \leqslant S^{\max}$, $E_z > 0$, $E_{zz} < 0$, $E(1) = 0$ and $E(0) = E^M > 0$. Solving backward for $S(t)$ we get

$$S(t) = S_0 e^{-\delta(t-t_0)} + \int_{t_0}^{t} E(1 - z(\tau))e^{-\delta(t-\tau)} d\tau. \tag{39}$$

The first term on the r.h.s. is a finite value that approaches zero as $(t - t_0)$ tends to $+\infty$, and the integral of the second term is a convergent one as long as the function $E(.)$ grows at most at a positive exponential rate lower than δ. In fact, the assumed more restrictive condition $E_{zz} < 0$ suffices to guarantee such a convergence of the integral, given the presence of an exponential discount term. Beyond this, since the model predicts that $z(t)$ is always chosen as a constant value, the above expression for $S(t)$ may be simplified to

$$S(t) = \left(S_0 - \frac{E(1-z)}{\delta}\right) e^{-\delta(t-t_0)} + \frac{E(1-z)}{\delta}, \tag{40}$$

where $\lim_{t\to\infty} S(t) \equiv S_\infty = \frac{E(1-z)}{\delta}$. Given S_0 and a constant value for z, the stock of pollutants always converges monotonically to a constant finite value, which is determined by emissions corresponding to such a value of the dirtiness index and the constant rate of natural regeneration δ. This result, however, does not suffices to prevent the environmental catastrophe. In our model, this situation may arise for a sufficiently high value of z given that $E_z > 0$ and S_∞ depends positively on z.

In particular, z could be chosen at a level for which emissions flow is exactly balanced out by the regeneration corresponding to the natural capacity of the environment to absorb pollution. In such a case, a steady state $\overset{\bullet}{S}(t) = 0$ arises from the beginning[17] and then $\forall t$

$$S(t) = \bar{S}_\infty = S_0 = \frac{E(1-\bar{z})}{\delta}. \tag{41}$$

Therefore, if $z < \bar{z}$ then $S(t)$ monotonically decreases below S_0 converging to a certain $S_\infty < S_0$, while if $z > \bar{z}$ then $S(t)$ monotonically increases converging to $S_\infty > S_0$. The no-catastrophe condition requires that $S(t) \leqslant S^{\max}$ for every $t \geq t_0$ and, in particular, that $S^*(t) \leqslant S^{\max}$, being $S^*(t)$ the socially optimal path for the stock of pollutants. This one emerges from the optimal balanced growth path, and is determined by substituting the value z^* into (40). Given monotonicity, the no-catastrophe condition is ensured when $S^*_\infty \leqslant S^{\max}$, where $S^*_\infty = \frac{E(1-z^*)}{\delta}$ is the limiting value for the stock of pollutants.

Let $z^{\max}$ be the value of $z(t)$ that eventually makes the stock of pollutants to catch up with the catastrophic level $S^{\max}$, and which satisfies $\delta S^{\max} = E(1 - z^{\max})$. Then, looking for a socially optimal positive balanced growth path that could be compatible with a non-catastrophic ecological state of the economy, we additionally need to impose

[17] Sometimes, the condition for stationarity has been taken as a condition for sustainability of the balanced growth path [Chevé (2000)]. This one may be considered as a very strong, near the consevationists, position where the stock of pollutants and other environmental variables remain constant while the rest of economic variables are still allowed to grow at a constant positive rate. However, as we will show immediately, it becomes excessive unless the upper ecological limit had been reached.

the parameter constraint

$$A\Phi - \alpha\rho + \alpha\Phi n \leqslant z^{\max}(A\Phi - \alpha A + \alpha\Phi A). \tag{42}$$

This condition, which corresponds to $z^* \leqslant z^{\max}$, is necessary and sufficient and also establishes the margins for sustainable optimal growth, given that $z^{\max}$ depends positively on the ecologically determined parameters $S^{\max}$ and δ. However, z^* is endogenously decided and, on the contrary, depends on preference and technological parameters. Thus, only if parameters determining the values of the variables along the balanced growth path satisfy condition (42), then the sustained socially optimal growth will be also ecologically sustainable in the long-run.

6.1 Optimal growth and environmental policy when we reach the limits of sustainability

Now, to complement the previous study of the sustainability of the long-run balanced growth path, we will derive the socially optimal behavior of the main variables in the model once the limits of the ecologically sustainable growth have been reached. Namely, the behavior of the economy when $S(t)$ catch up with the upper-limit $S^{\max}$ not eventually in the limit but at a finite moment in time. This case shows a great parallelism with the problem concerning the global warming of the Earth's surface and atmosphere caused by the anthropogenically enhanced greenhouse effect. It is well-known that the greenhouse gases accumulated in the atmosphere have reached the critical level which will lead to a catastrophic climate change. In this context, it is of great interest to know how can we manage efficiently this situation and what must we do, if we can do something, to guarantee a positive rate of growth in the long-run. The Kioto protocol, by imposing reductions on polluting emissions, represents a valuable instrument that arises from the necessary cooperating context and may contribute to resolve the previous problem.

According to this, we will study here the general optimal control solution when the non-negativity constraint, the control variable constraint and, specially, the state variable constraint are explicitly introduced into the dynamic optimization problem. This problem, which endogenizes both the local and aggregate externalities assuming that the central planner takes simultaneously into account the total costs and benefits from pollution abatement and the global negative effect of an ecological catastrophe, is formulated and solved in the Appendix. Next, we shall interpret the corresponding first order conditions.

Consider that $S(t^c) = S^{\max}$ for some $t^c > t_0$. This happens when the socially optimal value of the dirtiness index may be found on the interval $z^{\max} \leqslant z^* \leqslant 1$. According to what has been previously shown if $z^{\max} = z^*$ then $t^c = +\infty$, whereas if $z^{\max} < z^*$ then $t^c < +\infty$. Actually, only the latter becomes economically interesting at this point because in such a case the ecological limit to growth appears as a binding constraint in finite time.[18] Hence, using the first order conditions, we first deduce $\overset{\bullet}{\theta}(t^c) < 0$. Then, given that θ is not allowed to be negative, we conclude that $\theta(t^c) > 0$ and $E(1-z) = \delta S = \delta S^{\max}$. The latter implies that at t^c the dirtiness index, a variable which admits jumps, will take the value

$$z(t^c) \equiv z^c = 1 - E^{-1}(\delta S^{\max}) = z^{\max}, \tag{43}$$

irrespective of its previous value. This means that at t^c the central planner optimally decides a discrete and instantaneous change in the value of the dirtiness index from $z^* > z^{\max}$ to $z^c = z^{\max}$. Moreover, the strict inequality $z^c < 1$, which is required for a non-zero marginal utility of consumption, also implies $\eta(t^c) = 0$.

Therefore, the dynamic equation for s becomes $\overset{\bullet}{s} = -ns$, from which we get $s = s(t^c)\, e^{-n(t-t^c)}$ and $\overset{\bullet}{S} = 0$, which in turn implies that $\forall t > t^c$ $S = S^{\max}$, $\theta > 0$ and $\overset{\bullet}{\theta} < 0$. Thus, the dirtiness index will remain stuck to the value $z^c < 1$, and then $\eta = 0$, for all $t > t^c$. Moreover, the solution to the dynamic equation $\overset{\bullet}{\mu} = (\rho + \delta)\mu - \delta\theta e^{nt}$, starting from t^c, is $\mu = \mu(t^c)\, e^{(\rho+\delta)(t-t^c)} - \int_{t^c}^{t} \delta\theta(\tau)\, e^{n\tau} e^{(\rho+\delta)(t-\tau)} d\tau$. The dynamics of this shadow

[18] It is easy to show that

$$t^c = t_0 + \ln\left(\frac{\frac{E(1-z^*)}{\delta} - S_0}{\frac{E(1-z^*)}{\delta} - S^{\max}}\right)^{\frac{1}{\delta}},$$

which implies that the span of time elapsed before we reach the ecological limit, $T = t^c - t_0$, depends on the involved parameters according to $T\left(\overset{-}{z^*}, \overset{-}{S_0}, \overset{+}{S^{\max}}, \overset{-,+}{\delta}\right)$.

price couples to such of s, which is explained by population growth, and the transversality condition $\lim_{t\to\infty} e^{-(\rho-n)(t-t^c)}\mu s = 0$ implies $\mu(t^c) = \int_{t^c}^{\infty} \delta\theta(\tau)\, e^{n\tau} e^{-(\rho+\delta)(\tau-t^c)} d\tau$, where the integral is bounded because the integrand converges to zero, given that θ is positive but decreasing and $\rho > n$. As expected, now $\forall t \geqslant t^c$,

$$\mu(t) = \int_t^{\infty} \delta\theta(\tau)\, e^{n\tau} e^{-(\rho+\delta)(\tau-t)} d\tau \neq 0. \tag{44}$$

The particular dynamics for q, c and k arise from the solution to the dynamic system

$$q = (1-z^c)^{\alpha(1-\Phi)} c^{-\Phi}, \tag{45}$$

$$\dot{c} = \left(\frac{Az^c - \rho}{\Phi}\right) c, \tag{46}$$

$$\dot{k} = (Az^c - n)\, k - c, \tag{47}$$

with the initial condition $k(t^c)$ and the transversality condition

$$\lim_{t\to\infty} e^{-(\rho-n)(t-t^c)} c^{-\Phi} k = 0. \tag{48}$$

This dynamic system is similar in its structure to the standard AK model, but the constant marginal productivity of capital is now $Az^c < A$. Consequently, the particular solution is

$$k(t) = k(t^c) \exp\left\{\frac{Az^c - \rho}{\Phi}(t - t^c)\right\}, \tag{49}$$

$$\frac{c(t)}{k(t)} = \frac{\rho - Az^c + \Phi(Az^c - n)}{\Phi}, \tag{50}$$

$$q(t) = q(t^c) \exp\left\{-\left(Az^c - \rho\right)(t - t^c)\right\}, \tag{51}$$

$$q(t^c) = \left(\frac{\Phi\,(1-z^c)^{\frac{\alpha(1-\Phi)}{\Phi}}}{\rho - Az^c + \Phi(Az^c - n)}\right)^{\Phi} \frac{1}{k(t^c)^{\Phi}}, \tag{52}$$

$$\rho > Az^c(1-\Phi) + \Phi n, \tag{53}$$

$$y(t) = Ak(t^c) \exp\left\{\frac{Az^c - \rho}{\Phi}(t - t^c)\right\}, \tag{54}$$

$$\gamma_y(t) = \gamma_c(t) = \gamma_k(t) = \gamma^c = \frac{Az^c - \rho}{\Phi}. \tag{55}$$

These results completely characterize the economic system after period t^c, just on the limits of the ecological catastrophe with an accumulated stock of pollutants equal to $S^{\max}$ and the dirtiness index fixed at the level $z^c = z^{\max}$, which depends exclusively on the parameters of the emissions function (the rate of natural regeneration and the previous maximum stock) according to (43). Moreover, variables k, c and y grow at a common constant rate, which is positive under the assumption of a constant return to capital greater than the social discount rate, $r^c \equiv Az^c > \rho$. In this case, however, the socially optimal rate of growth and dirtiness index do not depend on the weight of environmental care in utility, α, or the population growth rate, n. Finally, according to the parameter constraint (53), which comes from the transversality condition (48), the ratio consumption to capital stock is constant and positive. Therefore, the model does not show transitional dynamics beyond t^c and all the endogenous variables conform again a unique socially optimal balanced growth path.

Along this balanced growth path, the rate of growth γ^c may be greater, equal or smaller than γ^* depending on the sign of the relationship between z^c and z^*. That is $\gamma^c \gtreqless \gamma^*$ if, and only if, $z^c \gtreqless z^*$. However, given that we have initially considered a high value of the optimal dirtiness index, sufficient enough to lead the economy close to the limits of the ecological catastrophe in finite time, and then it was optimally changed to a lower value z^c that ensures a constant aggregate stock of pollutants in the environment, we conclude that the rate of growth along this new path is smaller than the previous one.

As the previous results show, the emissions control and reduction recommended in the Kioto protocol seems to be a suitable policy if we want to avoid the ecological catastrophe. The evidence about the enhanced greenhouse effect and the global warming trend tells us that the critical level for the accumulated greenhouse gases in the atmosphere has been reached. Consequently, only coordinated actions involving governmental policies that stabilize and reduce the aggregate stock of such greenhouse effect pollutants may be effective at this stage. Moreover, the Kioto protocol also includes many other accompanying policies, which taken together may help to guarantee global efficiency and positive long-run growth. In short, we think that the proposals of this international treaty are essential to enable economic development to proceed in a sustainable manner.

From the socially optimal solution to the environmental problem compared with the decentralized one, we identify different opportunities for government interventions. First of all, an institutional one, which involves the government correcting the externalities associated with the local pollution abatement and the global environmental catastrophe. This may be done by setting the usual Pigouvian taxes and subsidies on pollution and abatement that make the competitive economy to work efficiently. Alternatively, the government may develop an allocative function, which implies a direct participation by means of a public abatement provision greater than the one decided in a decentralized economy. Moreover, if the optimal choice for economic variables lead the stock of pollutants to catch up with the critical upper-limit, then the government may impose abruptly cleaner technologies by increasing abatement suddenly, which will reduce pollution standards and control for the stock of pollutants in a short period of time. This will just make stationary or even to decrease the stock of pollutants, which is needed to avoid the ecological catastrophe. On the other hand, the government may implement indirect environmental policies such as information or awareness campaigns [Chevé (2000)]. These ones influence social preferences for environmental conservation, the environmental willingness to pay and, hence, the demand for environmental quality. In other words, participation can make people more environmentally conscious and may prevent that the environment be felt as an obstacle to growth [Bimonte (2001)]. Finally, the government may put into action population controls and other development encouraging measures that accelerate the demographic transition. This is important because of the influence of population growth on the environmental quality and the long-run rate of growth.

7 Conclusions

In this paper, we have built a general equilibrium one-sector endogenous growth model in which pollution is a by-product of economic activity but it may be reduced by spending a fraction of the aggregate output on abatement. We identify two types of externalities.

First, a local pollution externality associated with the emissions flow and abatement activities. Second, an aggregate externality which comes from the fact that people take into account the local negative effect from pollution, but they are not aware of the global negative effect from the accumulated stock of pollutants. To tackle this problem, which has a great parallelism with the problem of the global warming of the Earth's surface and atmosphere caused by the anthropogenically enhanced greenhouse effect, we introduce an absolute upper-limit to the accumulated stock of pollutants beyond which we fall in an ecological catastrophe. First of all, we study the decentralized competitive economy. The equilibrium path is not Pareto-optimal and sustained growth is not sustainable, leading the economy to an environmental catastrophe. Consequently, we move to study the socially optimal equilibrium.

We have proved that the optimal path does exists, it is unique, and does not show transitional dynamics. We found that the rate of growth depends negatively on the weight of environmental care in utility and positively on the population growth rate. Moreover, the latter effect is stronger as higher is the weight of environment in the utility function. We also found a trade-off between growth and environmental quality, which results from agent decisions, because increased abatement effort crowds out resources from capital accumulation and growth. That is, the higher the rate of growth the higher the dirtiness index associated with production. However, this is not a problem at least until the stock of pollutants catch up with the ecological critical level.

We have analyzed the opportunities for an efficient management of the economy with special attention to the potential risk of environmental collapse. In the context of our model, the usual efficiency condition is not sufficient for sustainability but, as we have shown, Pareto optimality is necessary to produce sustainable outcomes. We have identified conditions for sustainability of the optimal balanced growth path. However, if the stock of pollution corresponding to the optimal path reaches the upper-limit, the central planner could still change the value of the dirtiness index by increasing drastically abatement activity in line with the proposals of the Kioto protocol. Obviously, this means that even in the aftermath sustainability may be guaranteed but at the cost of a lower rate of growth.

8 Appendix

Consider the more general optimal control problem involving either non-negativity constraints, pure control variable constraints and pure state-space constraints, applied to our endogenous growth model with environmental concern and awareness. In particular, such constraints are: (i) $z(t) \geqslant 0$, (ii) $1 - z(t) \geqslant 0$ and (iii) $S(t) \leqslant S^{\max}$, which add to the usual dynamic and boundary constraints for k and S.

The third type of constraint in the above classification consists of one constraint in which no control variables are present. This constraint places a restriction on the state space, delimitating the permissible area for the accumulated stock of pollutants. Writing the current stock in per capita terms we get $S^{\max} - e^{nt}s(t) \geqslant 0$. However, given that $e^{nt}s(t)$ is not allowed to exceed $S^{\max}$, when the constraint is binding, i.e. $S(t) = S^{\max}$, we impose the new condition

$$\frac{d\left(e^{nt}s(t)\right)}{dt} = e^{nt}\left[e\left(1 - z(t)\right) - \delta s(t)\right] \leqslant 0 \quad \left(\text{whenever } e^{nt}s(t) = S^{\max}\right),$$

where we have made use of the equation representing the motion of the stock of pollutants in per capita terms $\overset{\bullet}{s}(t) = e\left(1 - z(t)\right) - (\delta + n)\,s(t)$. Then, the current value Hamiltonian after writing all the variables in per capita terms is

$$\underset{\{c,z,q,k,\mu,s,\eta,\theta\}}{H^{c}} = \frac{c^{1-\Phi}\left(1 - z\right)^{\alpha(1-\Phi)}}{1 - \Phi} + q\left[Akz - c - nk\right] + \mu\left[e(1 - z) - (\delta + n)\,s\right] +$$

$$+\eta\left[1 - z\right] - \theta e^{nt}\left[e(1 - z) - \delta s\right].$$

Here, q and μ are the co-states for k and s respectively, and η and θ are Lagrangian multipliers associated with the control variable constraint and the state variable constraint respectively. Both η and θ are dynamic multipliers because their corresponding constraints must be satisfied at every period t. Given that the control inequality constraint is linear, the first order necessary conditions arising from Pontryagin's principle and Kuhn-Tucker theorem are

$$q = c^{-\Phi}\left(1-z\right)^{\alpha(1-\Phi)},$$

$$qAk + \mu e_z - \eta - \theta e^{nt} e_z - \frac{\alpha c^{1-\Phi}\left(1-z\right)^{\alpha(1-\Phi)}}{1-z} \leqslant 0,$$

$$z \geqslant 0, \quad z\left[qAk + \mu e_z - \eta - \theta e^{nt} e_z - \tfrac{\alpha c^{1-\Phi}(1-z)^{\alpha(1-\Phi)}}{1-z}\right] = 0,$$

$$\overset{\bullet}{k} = Akz - c - nk,$$

$$\overset{\bullet}{q} = \rho q - Azq,$$

$$\overset{\bullet}{s} = e(1-z) - (\delta + n)\, s,$$

$$\overset{\bullet}{\mu} = (\rho + \delta)\, \mu - \delta\theta e^{nt},$$

$$1 - z \geqslant 0, \quad \eta \geqslant 0, \quad \eta\,(1-z) = 0,$$

$$e(1-z) - \delta s \leqslant 0, \quad \theta \geqslant 0, \quad \theta\left[e(1-z) - \delta s\right] = 0.$$

To make clear that the three latter rows of first order conditions only apply when $e^{nt} s\,(t) = S^{\max}$, we append the complementary-slackness condition and the restriction on the way θ changes over time

$$S \leqslant S^{\max}, \quad \theta\left[S - S^{\max}\right] = 0, \quad \overset{\bullet}{\theta} \leqslant 0 \quad \left(= 0 \text{ when } S < S^{\max}\right).$$

Finally, we also need the initial conditions k_0 and s_0 and the transversality conditions

$$\lim_{t\to\infty} e^{-(\rho-n)(t-t_0)} qk = 0,$$

$$\lim_{t\to\infty} e^{-(\rho-n)(t-t_0)} \mu s = 0.$$

These necessary conditions are also sufficient for a maximum because the Hamiltonian function satisfies the required concavity conditions. It is easy to see that, when the constraints mentioned at the beginning of this appendix are nonbinding, the previous first order conditions reduce to the ones studied in section 4, giving only interior solutions. However, if anyone of such constraints changes its status from nonbinding to binding, then the previous first order conditions become fully operative and corner solutions are also feasible. This is the case analyzed in section 6 with special regard to the pure state-space constraint.

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